SOLVING CONFORMAL CONTACT PROBLEMS

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Abstract

The stresses between railway wheels and rails can be computed using different types of contact models: simplified methods, half-space based boundary element approaches and finite element models. For conformal contact situations, particularly the contact between flange root and rail gauge corner, none of these models work satisfactorily. Finite element methods are too slow, half-space approaches ignore the effects of conformality, and simplified approaches schematize the elasticity of the material even further.

In this paper we present a thorough investigation of the conformal contact problem. We use CONTACT’s boundary element approach together with numerical influence coefficients, that are computed using the finite element approach. The resulting method is fast and detailed and can be embedded in vehicle system dynamics simulation. With respect to the half-space approach, the contact becomes longer and narrower and the area and normal force $F_n$ reduce. In our experiments, the maximum pressure increased 30%. Finally the longitudinal and lateral forces changed up to 15% of the maximum $\mu F_n$.

1. INTRODUCTION

Each time we write a paper about the CONTACT model (e.g. [15, 16, 17]), we are confronted with the assumptions underlying the half-space approach [5, 7, 11]:

1. the characteristic sizes of the bodies in contact are large compared to the size of the contact patch;
2. the bodies are made of homogeneous, isotropic, linearly elastic materials;
3. deformations and strains are small, and effects of inertia can be ignored.

In practice we often find situations where these assumptions are violated. Particularly for the first two of them, regarding the flatness of the geometry and plastic deformation of the material. Of these two aspects, the effect of plasticity is already covered by many publications in the literature, e.g. [3, 13, 18]. Much less is known about conformal contact situations, in which the contact patch is curved. A few studies can be found, e.g. [1, 2], the most comprehensive one being the PhD thesis of Li [9]. But even that does not answer the question how large the effect is in different circumstances. Therefore we don’t know the implications for vehicle behaviour (derailment scenarios) and profile design (spreading the load over sufficient area), and we don’t know the effects of conformality on the mechanisms leading to RCF (costs of grinding and replacing rails). In this paper we present our research into solving conformal contact problems (the methodology that is used and its results), hereby we aim at filling these gaps.

2. ANALYSIS OF CONFORMAL CONTACT SITUATIONS

A prototypical situation where conformal contact analysis is needed is shown in Figure 1. This figure shows a profile that is measured for a piece of heavily worn rail. The section is taken from a radius 2000 m curve. The original profile is 54E1 and the rail steel is R260Mn (nowadays, R350Ht is used in such curves). The inset in the figure shows the computed radius of curvature at the rail gauge corner. The value goes down to less than 10 mm at an angle $\delta=45^\circ$ with respect to the vertical direction. At this radius, the normal direction changes $40^\circ$ in a distance of just 7 mm, a typical size for the contact patch. In this region, the flatness assumption is violated, such that it is not appropriate to schematize the bodies as a half-space anymore. Instead, the curvature and wedge-shape as schematized by the dashed red line should be used.

The effects of this are multiple:
1. There is not a single plane that the contact area and surface stresses can be projected upon, hence there are no global “normal” and “tangential” forces anymore;
2. With the direction of the normal vector, the local conicity changes too. Therefore, the spin creepage is no longer constant but varies throughout the contact patch;
3. The indicated wedge-shaped geometry will have larger elastic deformations when subjected to a load than the elastic half-space. This difference increases with decreasing opening angle;
4. The other way around, the corresponding wheel geometry will have smaller elastic deformations than the elastic half-space. This difference leads to coupling between the normal and tangential stress components.

We present an approach in which these aspects are all addressed. This tells us firstly how we can compute the “true” solution in great detail and secondly how it deviates from approximate solutions that can be computed easily. This allows us to tell then when we need the full approach versus when simplifications can be justified.

3. METHODOLOGIES FOR SOLVING CONTACT PROBLEMS

One might think that conformal contact situations can be analyzed using the finite elements method (e.g. [8]). However, this is far from trivial. The main difficulties come from the large motions of material points of the wheel. In the standard methodology, the finite element mesh is attached to these material points. Because of this, it is difficult to compute steady state rolling scenarios. One typical approach that is used for this is to let the wheel roll for a specified distance in order to let a steady state set in, and then use a refined region to compute the final result [18]. This needs many hours of calculation time for computing a single scenario, and then there are still methodological difficulties.

- It is hard to assess the numerical quality of the results, because the mesh cannot be refined easily;
- It is difficult to separate physical effects from numerical artefacts, e.g. remaining deviations from the steady state, effects of boundary conditions, effects of the mesh that is used, etc.

An alternative that seems worth-while is the “arbitrary Lagrangian-Eulerian” (ALE) approach [10]. This uses a fixed mesh with increased resolution near the contact zone, and computes the flow of material through the mesh. Again there are difficulties to overcome.

- The ALE approach uses “advection” to describe the flow of material. This is a tricky operator that needs careful consideration in the discretization approach (e.g. using an upstream approach);
- The contact conditions are implemented in [10] (and other finite element packages) using a penalty approach. This means that the contact conditions are not precisely satisfied, making it difficult to analyze what is really going on inside the contact patch.

A disadvantage that pertains to these all-FEM approaches is that large parts of the system are discretized using a mesh. In practice, many deformations in the system are of less concern than global body motions. This can be seen for instance in Kaiser’s results for the effect of axle flexibility on the contact results [6]. An alternative approach is therefore to start from multibody simulation and introduce flexible bodies therein [4]. The multibody software computes the overall positions and motion of the system components. For this it needs the forces acting between wheel and rail, which are typically computed using a simplified contact algorithm. The two parts are then run...
alternatingly. The MBS software determines the overall contact geometry, and the contact algorithm computes deformations and the resulting contact force.

In between simplified algorithms and the finite element method is the approach that is followed in our CONTACT software [14]. The advantages for conformal contact are that it is fast and can be embedded in MBS, and that it persistently solves the shear stresses in full detail.

1. The main ingredient is to separate two problems: to find the stresses acting locally between the contacting bodies, versus the global response (deflections) of the two bodies as a whole.

2. A second ingredient is to use influence coefficients and a superposition principle. The response of the two bodies to unit loads on their boundary elements is computed and stored, then the proper superposition of these loads is sought such that the contact conditions are satisfied.

3. The third ingredient that is used in CONTACT is to resort to the half-space approach. Instead of the response of the true bodies we take the influence coefficients for an idealized half-space, which are known analytically [5, 7].

The essence of the half-space approach is that for massive elastic bodies, the stresses and deformations decay rapidly with the distance to the contact area. Therefore the contact zone does not “feel” any influence of the geometry farther away; all massive bodies respond the same to concentrated loading on their surfaces, and this response is the same as for an elastic half-space.

In case of a surface with significant curvature in the contact region we cannot speak of a massive body anymore. The response to surface loading is different. This is dealt with by replacing ingredient 3. above by the following alternative:

3’. We compute the response of the true geometry due to unit loads numerically using finite elements. The resulting numerical influence coefficients are then imported into CONTACT and used in our detailed contact analysis.

4. NUMERICAL CALCULATION OF INFLUENCE COEFFICIENTS

4.1 Numerical influence coefficients for the half-space

A key step in our investigation is to compute the influence coefficients for the half-space numerically. This may seem illogical because these influence coefficients are already known analytically. But there is a huge benefit in this step. It allows us to verify our approach systematically. Among others, this reveals the interaction between local and global deformation modes. This interaction is typically overlooked, and then hampers the combination of MBS with detailed contact approaches.

We consider a large rectangular block of homogeneous linearly elastic material, that occupies the domain \((x,y,z) \in [-L,L] \times [-L,L] \times [-L,0] \text{ mm}\). The block is assumed fixed at the far sides \(x=\pm L, y=\pm L\) and \(z=-L\). The upper surface is free to move and free of stress, except for a central square element of size \(\delta x \times \delta y\) where a unit stress \(p_x=1, p_y=1\) or \(p_z=1\) N/mm\(^2\) is applied. We compute the displacements \((u_x,u_y,u_z)\) at the surface at regularly spaced points. These are the influence coefficients that are needed in the contact algorithm.

Figure 2 – Illustration of different meshing strategies. Left, middle: ANSYS; right: SEPRAN mesh.
Different meshing strategies have been tried as illustrated in Figure 2. We started using ANSYS at first, later switched to the SEPRAN package (originally developed at Delft University of Technology), because of its superior control over the mesh generation. In any case symmetries and anti-symmetries in the planes $Oxz$ and $Oyz$ are exploited such that one quarter needs to be computed. Linear basis functions did not give very good results, but quadratic elements worked fine. The element sizes could be increased rapidly towards the fixed sides of the block, e.g. doubling the mesh size for each next element as shown in Figure 2 (right). Unsurprisingly, we succeeded in computing the displacements within 1% accuracy of the theoretical result. More important are other findings from this experiment:

- The finite element model cannot deal well with the piecewise constant loading that is used, exhibits under- and overshoots at the edges of the loaded element. This is not relevant for our purpose, because these inaccuracies are sub-grid with respect to where the influence coefficients are required.
- Except for these under- and overshoots, the error looks like a constant offset with respect to the analytical result. This offset error is roughly halved each time the domain size is doubled. In order to bring it down to a small level, a huge domain size is required.

4.2 Propagation of errors in influence coefficients

As a next step, the effect that the inaccuracies in the numerical influence coefficients have on the contact results is assessed. How do the inaccuracies propagate? What size of inaccuracies can be tolerated in which circumstances?

These questions are investigated in two different ways. First we computed the analytical influence coefficients and modified these in a structured way. Either by adding an offset error or by ignoring the influence beyond a certain distance. In the second approach the actual influence coefficients computed by SEPRAN were used.

A simple test case with Hertzian contact is used with circular contact with semi-axes $a=b=4$ mm and vertical load $F_x=20$ kN. The corresponding wheel radius $R_{ax}$ and transverse radius $R_t$ are both 486 mm. The contact problem is discretized with elements of $\Delta x=0.4$, 0.2 and 0.1 mm ($20\times20$ to $80\times80$ elements). For each grid we computed the solution with true influence coefficients and with several sets of perturbed influence coefficients. For the coarsest grid, problem is also solved with influence coefficients computed with SEPRAN.

![Image 1](image1.png)

**Figure 3 – Left:** effect of “truncation” of the influence coefficients. **Right:** Accuracy of total forces $F_n$, $F_t$ for influence coefficients computed using different SEPRAN meshes.

Figure 3 (left) shows one result of what happens when the influence between surface points is truncated. Instead of the typical Hertzian pressure distribution, the pressures show a dip at the center of the contact area. This behaviour can be understood easily. The points at the outer ring of the contact area don’t feel the influence of the points at the other side anymore. Consequently higher pressures are required to keep the surfaces from penetrating. This makes it easier for the points at the center of the contact area: they get larger displacements already from other points, and therefore need lower pressures themselves to satisfy the contact constraint.

This test simply shows that truncation of influence coefficients should be avoided as much as possible, which can typically be done easily. More important is then the next test, in which the accuracy of the forces is investigated. The main result on this is that offset errors propagate strongly to the contact results, which is demonstrated in Figure 3, right. The total normal force $F_n$ is increased by using numerical influence coefficients: if all the deformations are underestimated a little, then more pressure is needed on the surfaces to make them fit. To bring
the error down to 5%, the domain size $L$ used in SEPRAN must be larger than 50mm, and using larger domains allows to bring the difference down further.

This test provides important information on how the results of CONTACT should be understood. Only local deformations are involved. Global deflections should be computed separately (using flexible modes in MBS) and be included in the “undeformed” geometry.

Note that global and local deformations can not be separated well when the true rail geometry is discretized using a finite element approach. If the finite element results are then combined with a flexible MBS approach, the same deformations may be accounted for twice, such that a wrong contact stiffness is used.

4.3 Influence coefficients for the quasi quarter-space

Based on the previous results we are ready to compute numerically the influence coefficients for conformal contact geometries. Following Li [9], the influence coefficients are computed for the quasi quarter-space. A true quarter-space consists for instance of the first quadrant in $\mathbb{R}^2$ or two octants in $\mathbb{R}^3$. The name quasi quarter-space is used for geometries with rounded corner, and with opening angles that can be different than 90°. The results in this paper concern the worn profile of Figure 1, with radius of curvature $R_w=10$ mm near the origin and opening angle 129°, and a matching wheel profile with concave surface with $R_n=-10$ mm and angle 231°.

$\begin{align*}
\delta_x &= A(ix, is, jx, js) \\
\delta_s &= B(is, js) \\
\delta_n &= C(is, js)
\end{align*}$

Generalized coordinates $(x,s,n)$ are introduced besides the Cartesian coordinates $(x,y,z)$. Here $x$ is the rolling direction, $s$ is the lateral position measured along the surface relative to a reference point, chosen at $\delta=45^\circ$ in Figure 1, and $n$ is in the direction of the local surface normal $n$. A mesh is defined for the contact calculation using $m_x mx, m_y$ rectangular elements of size $\delta x \times \delta s$. The full matrix of influence coefficients consists of a 4D array $A\{ix, is, jx, js\}$. Each element of this matrix consists of $3 \times 3$ coefficients, expressing the displacements $(u_x, u_y, u_z)$ at location $(x_w, s_w)$ due to unit loads $(p_x, p_y, p_z)$ at $(x_p, s_p)$. For a constant profile in $x$-direction, all coefficients with the same offset $i-j$ are equal such that one dimension can be dropped.

Consequently we must solve $6 m_y$ problems with SEPRAN in order to fill $A$ (placing unit loads $p_x=1, p_y=1$ or $p_z=1$ at the wheel and rail surface at all elements along one row of the potential contact), requiring 1—30 min each, depending on the desired accuracy. This is achieved by a script that generates optimized meshes around each loaded element (Figure 4) and by converting the load from $(x,s,n)$ to $(x,y,z)$ coordinates. The resulting displacements are converted back to $(u_x, u_y, u_z)$ and interpolated to the positions of the mesh for the contact problem. Finally they are stored in a file and then incorporated in the CONTACT program.

5. UNDEFORMED DISTANCE AND CREEPAGE

You can beat your brains many times on the computation of the undeformed distance between the surfaces and their relative motion, the creepages. Consider for instance the projection of the path of a wheel surface particle onto the curved rail surface. How does it move, and what’s the velocity? We present a relatively simple approach with clearly prescribed steps that leads to the desired result.

We start by selecting the “reference contact point”. This can be the point with maximum penetration of the two surfaces or any other convenient point near the center of the contact area. From this point we automatically get a
“reference normal direction”. This can be used for rotation of the total forces from vertical/lateral to “normal/tangential”, even though the meaning of these remain less well defined than in the planar contact case.

Next, the desired contact grid can be laid out on both surfaces, with fixed grid spacings $\delta x$ and $\delta s$. Even though the surfaces are curved, points with the same s-coordinate will be close together and will have almost opposite normal direction w.r.t. the $(s,n)$-plane (Figure 5). It is therefore appropriate to define the curved contact plane by the mean position of the two surfaces, measure the distance between points with the same coordinate $s_j$ and use this as undeformed distance $h(x,s_j)$.

For the overall approach of the bodies, i.e. the maximum penetration of the undeformed surfaces, it is important to realize that it is oriented in reference normal direction. This approach then consists of $(s,n)$-components $(0, h_0)$ at the reference contact point, and $(-h_0 \sin(\delta_0 - \delta), h_0 \cos(\delta_0 - \delta))$ at other points with contact angle $\delta$ instead of $\delta_0$. An approach in normal direction at the reference point thus implies also a tangential shift at points with different orientation.

Next we turn to the computation of the rigid slip or creepage between the two surfaces. Different presentations on this can be found in the literature, including to some extent the influence of curvature, see e.g. [7, 9] or [12]. These presentations are not so easy to understand, and therefore it’s not easy to tell how they should be used or extended for different circumstances. A new insightful derivation is obtained with the help of Figure 6.

We consider a wheelset placed level (zero roll angle) with yaw angle $\psi$ on a tangent track. Now consider the wheelset rotating about its axle in place, leaving the forward velocity out of the picture for a while. The dashed line in Figure 6 indicates a vertical plane perpendicular to the wheelset axle. This plane is shown on the right, including the path of particles on the wheel surface. This path is parameterized as $x=x_0 - R_0 \omega t$, $y=y_0 - R_0 \omega \psi t$ and $z=z_0 + R_0^2 \omega^2 t^2/2$. This uses that $\psi$ is a small angle and approximates the circular arc using a parabola. From this parameterization we get the particle velocity as $(-R_0 \omega, -R_0 \omega \psi, \omega(x_0-x))$.

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**Figure 5** – Schematic illustration of undeformed distance computation, measuring the distance between points with equal s-coordinates.

**Figure 6** – Schematic illustration of the path of a wheel surface particle, which is parameterized easily as a function of time $t$. 
The rigid slip in x-direction consists of the difference between forward and circumferential velocity, \( V = oR \). This is split into the reference value \( \xi = V = oR_0 \) plus the effect of the rolling radius \( o(R - R_0) \). This last term can be expanded further as \( o((s_0 - s)\sin(\delta_0) + B(s)) \). The first part of this corresponds to the spin creepage \( \eta \) that is well known from planar contact situations; the latter part is the contribution of curvature of the contact plane.

For the lateral component of the rigid slip, the key is to realize that the change of vertical coordinate \( z \) of a wheel particle also changes its lateral coordinate \( s \). We must transform the velocities to \( (x, s, n) \)-coordinates! Expressing the \( s \)-position of a particle as function of \( y \) and \( z \) we get

\[
\frac{ds(t)}{dt} = \frac{ds(y(t), z(t))}{dt} = \frac{\partial s}{\partial y} \frac{dy}{dt} + \frac{\partial s}{\partial z} \frac{dz}{dt} = -R o\psi \cos(\varphi) + o(x_0 - x)\sin(\delta)
\]

The first term corresponds to the lateral creepage \( \eta \), the second one to the spin creepage term \( \varphi x \). Note that both of these depend on the contact angle \( \delta \), such that both of them vary with the curvature of the contact patch.

### 6. RESULTS

#### 6.1 Influence coefficients for the quasi quarter-space

![Numerically computed influence coefficients](image_url)

Figure 7 – Numerically computed influence coefficients for different geometries. Left: \( A_{\delta x}^{rail} \), lateral deformation due to lateral load for the rail. Middle and right: \( A_{\delta s}^{rail} \) and \( A_{\delta s}^{wheel} \), lateral deformation due to normal load for the rail and the wheel.

Many interesting pictures can be presented for the influence coefficients for the quasi quarter-space. A sample of these results is presented in Figure 7. These pictures show the deformations due to unit loads at the central element \( \delta s = 0.4 \text{ mm} \) around \( s = 0 \) (contact angle \( 45^\circ \)). Different lines are for different radii of curvature: varying from \( R = 10, 30, 80 \) to \( R = 300 \text{ mm} \) and \( R = \infty \), i.e. the half-space results.

The picture on the left shows the lateral displacements \( u_s \) under a unit lateral load \( p_s = 1 \) in the loaded element. This shows that the rail surface displaces more if the radius \( R \) is reduced, i.e. the effective lateral stiffness of the rail is reduced. At the same time the effective stiffness for the wheel increases a bit. These effects partially cancel each other, but not entirely. Therefore the displacements become slightly larger at the same load when conformality is included.

The middle and right pictures show the lateral displacements due to a unit normal load \( p_n = 1 \) at the central element. In the half-space, the surface is drawn towards the point where the load is applied. This remains so for the wheel (right picture), where the amount of lateral displacement actually increases with reducing radius of curvature. For the rail the situation is opposite (middle picture). Here the changing normal direction causes displacements to be reduced and even change sign at a small distance from the loaded element. When the displacements for wheel and rail are combined, the two effects are reinforcing instead of opposing each other. As a consequence, normal pressures invoke a tangential displacement difference, which causes tangential stress to come about.

#### 6.2 Effect on the total force

Finally we come to the main results of this article, concerning the effect of curvature and conformality on the contact results. For this we consider a scenario with the contact takes place at the transition from tread to flange contact, at a contact angle of \( 45^\circ \) (reference normal direction). This defines the spin creepage \( \varphi = -0.001537 \) rad/mm for a planar contact analysis. The longitudinal and lateral creepages \( \zeta \) and \( \eta \) can take any value though,
Depending on the angle of attack (yaw angle \(\psi\)) and the rolling radius difference between left and right wheels of the wheelset. These depend among others on the radius of the curve and the steering ability of the vehicle.

The curvature of the contact patch is defined through the transverse radius \(R_w = 10\) mm for the rail profile. For the conformality two different values are used: a highly conformal scenario with \(R_w = -10.5\) mm (worn wheel) and a modest conformity with \(R_w = -13.0\) mm, which occurs in the flange root of a new wheel with S1002 profile. Two different typical values are also used for the wheel load, viz. 20 kN and 100 kN. Corresponding values for the approach \(h_0\) are derived using the Hertzian theory. This approach is then held fixed in the simulations such that the total forces become variable again.

The results of these tests are summarized as follows.

- Including conformity reduces the width of the contact patch in lateral direction (Hertzian ellipse: semi-axis \(a\)) and increases the length in rolling direction (\(b\)). At the same time the area of the patch decreases by 30–35\%, and the maximum pressures increase correspondingly (Table 1). This is mainly because the approximation of the undeformed distance by a quadratic function and consequently the Hertzian theory break down at high curvature.

- The normal force (in reference normal direction) deviates 10–15\% between the planar and conformal contact analyses (Table 1). In vehicle dynamics simulations one may eventually see the same force come about, but then the approach and contact stiffness have changed. The largest effect in this comes from the undeformed distance calculation. Moreover, the normal force is no longer a function of the penetration alone but is affected by the creepage too. This is because of the coupling between normal and tangential stresses in the numerical influence coefficients. The effect of this is limited to about 5\%.

- In situations with large creepage the tangential forces are saturated such that the difference in the results is dominated by the differences in \(F_n\), see the force map in Figure 8. At small creepages, marked differences can also be found due to the space-variation of the creepages, incorporating the curvature in the local rolling radius \(R(s)\) and local contact angle \(\delta(s)\). A third effect comes from using the true influence coefficients instead of using the half-space approach. The size of these effects varies with the creep situation as illustrated in Figure 9. The difference in forces \(F_n\) and \(F_t\) is shown in the traditional way in Figure 10 (left), whereas Figure 10 (right) shows the spatial distribution for a single case.

The tests described here involve 800 data points for each method and each scenario. These could be computed within a few hours, i.e. using on average less than a second per case.

### 7. SUMMARY AND DISCUSSION

In this work we presented our approach for solving conformal contact problems. This consists of CONTACT’s boundary element method, using superposition of basic solutions, together with numerically computed influence coefficients. The main strengths of this approach are that it re-uses CONTACT’s proven algorithms for rolling contact, strictly obeying Coulomb’s friction law or even more elaborate conditions everywhere in the contact interface, and that it delivers results fast and can be embedded in a multi-body approach.
Figure 8—Left: Map of tangential forces ($F_x$, $F_y$) as function of longitudinal creepage $\xi$ and yaw angle $\psi$ for test 4 ($R_y$=−13.0 mm, $F_n$=100 kN). Right: difference between Hertzian and conformal results.

Figure 9—Left: contribution of the creep variation to the results. Right: contribution of numerical influence coefficients.

Figure 10—Left: tangential forces $F_x$ and $F_y$ for three values of the creepage $\xi$. Right: picture of the contact area and tangential traction distribution for Hertzian and conformal approaches ($\xi$=−0.8%, $\psi$=14 mrad).

Particular attention is paid to the separation of local deformations and global deflections of the wheel and rail. This makes our approach particularly well suited for combination in MBS. We believe that this will be an important advantage compared to all-FEM approaches.

The importance of conformal contact modelling is illustrated by initial experiments for an actual worn rail profile. Considerable differences were found in realistic situations that are in no way extreme. Therefore it is advised to take these influences into account in detailed investigations of derailment scenarios, profile design, and in the study of RCF and wear.
Further research is required to determine the size of the effects in a wider range of circumstances. Another suggestion is to use our approach as a benchmark for tuning and validating simplified contact algorithms for conformal contact. Finally our approach can be extended to incorporate plasticity, by further integration of our contact algorithm with finite element modelling.

References


